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Viscous heating in liquid flows in micro-channels

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Abstract

Many experimental works on forced convection through micro-channels evidenced that when the hydraulic diameter is less than 1 mm, conventional theory can no longer be considered as suitable to predict the pressure drop and convective heat transfer coefficients. This conclusion seemed valid for both gas and liquid flows. Sometimes the authors justified this claim by invoking "new" micro-effects. On the contrary, in this paper the explanation of the experimental results obtained for micro-channels in terms of friction factors will be researched inside the conventional theory (Navier–Stokes equations). In particular, this paper will focus on the role of viscous heating in fluids flowing through micro-channels. A criterion will be presented to draw the limit of significance for viscous dissipation effects in micro-channel flows. The role of the cross-sectional geometry on viscous dissipation will be highlighted and the minimum Reynolds number for which viscous dissipation effects can no longer be neglected will be calculated as a function of the hydraulic diameter and of the micro-channel geometry for different fluids. It will be demonstrated how viscous effects can explain some experimental results on the Poiseuille numbers in micro-channels, which recently appeared in the open literature.

Keywords: Micro-channels; Scaling effects; Viscous dissipation; Laminar regime; Friction factors

1. Introduction

Micro-flow devices (MFDs) are downscaled devices like channels, nozzles, diffusers, pumps, mixers, heat pipes, sensors, transducers and actuators that are incorporated in complex systems for medical diagnosis and surgery, chemical analysis, biotechnology and motor management. The development of micro-fluidic devices during the past 10 years has been particularly striking. Today, the research on micro-electro-mechanical systems (MEMS) is exploring different applications that involve the dynamics of fluids and the single and two-phase forced convective heat transfer in micro-channels. Recent advances in micro-fabrication techniques make it possible to build micro-channels with very small hydraulic diameters (100–0.1 μm) using different methods. For example, chemical etching is used to build micro-channels on silicon wafers; in this case, the shape of the channels depends on a variety of factors such as the crystallographic nature of the silicon used. When a KOH-anisotropic etching technique is employed, it is possible to obtain micro-channels having a fixed cross-section that depends on the orientation of the silicon crystal planes; for instance, micro-channels etched in ⟨100⟩ or in ⟨110⟩ silicon, will have a trapezoidal cross-section (with an apex angle of 54.74° imposed by

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Nomenclature

a, b	maximum and minimum width of the cross-section, m
c_p	fluid specific heat, J kg^{-1} K^{-1}
D_h	hydraulic diameter of the duct ($=4\Gamma/\Omega$), m
Ec	Eckert number, ($=W^2/(2c_p\Delta\theta_{ref})$)
f	Fanning friction factor
h	cross-section height, m
k	fluid thermal conductivity, W m^{-1} K^{-1}
L	micro-channel length, m
L^*	dimensionless micro-channel length ($=L/D_h$)
Ma	Mach number ($=W/c$ with c = acoustic velocity)
p	pressure of the fluid in the duct, Pa
p^*	dimensionless pressure defined in Eq. (4)
Pe	Peclet number ($=Re\,Pr$)
Pr	Prandtl number ($=\nu/\alpha$)
q	linear heat flux, W m^{-1}
Re	Reynolds number ($=WD_h/\nu$)
$T(\cdot)$	dimensionless fluid temperature
$u(\cdot)$	axial fluid velocity, m s^{-1}
$V(\cdot)$	dimensionless fluid velocity
W	fluid average velocity, m s^{-1}
x, y, z	dimensionless Cartesian co-ordinates

Greek symbols

α	thermal diffusivity ($=k/\rho c_p$), m^2 s^{-1}
Φ^*	dimensionless viscous-energy-dissipation defined by Eq. (6)
γ	micro-channel aspect ratio ($=h/a$)
Γ	wetted perimeter, m
Γ^*	dimensionless perimeter ($=\Gamma/D_h$)
ν	kinematic viscosity, m^2 s^{-1}
μ	dynamic viscosity, kg m^{-1} s^{-1}
ρ	fluid density, kg m^{-3}
θ	fluid temperature, K
Ω	cross-section area, m^2
Ω^*	dimensionless cross-section ($=\Omega/D_h^2$)
ξ, η, ζ	Cartesian co-ordinates, m

Subscripts

app	apparent
v	mixing cup (or bulk)
exp	experimental
in	inlet
ref	reference
th	theoretical
w	wall

the crystallographic morphology of the silicon) or a rectangular cross-section respectively. A vast amount of experimental and numerical studies on the pressure drop and the convective heat transfer in micro-channels have appeared in the open literature for these geometries; a complete review of these works is given by Morini [1]. Many authors have suggested new correlations in order to predict the friction factors and the convective heat transfer coefficients in micro-channels both in the laminar and turbulent regimes. Unfortunately, the new correlations proposed to predict the Nusselt number in micro-channels disagree with one another, as demonstrated by Owhaib and Palm [2] and each correlation disagrees with the conventional prediction.

In order to explain this behaviour many researchers claimed that the Navier–Stokes equations are no longer adequate to study flows through micro-channels because the characteristic lengths of MFDs are sometimes of the same order of magnitude as the mean free path of the molecules of the flowing fluid; this occurs especially for gas flows. On the contrary, for liquids with a distance between the molecules much smaller than that for gases, the continuum approach and the Navier–Stokes equations hold even in micro-channels.

Herwig and Hausner [3] observed that in order to study the forced convection of liquids in the laminar regime a common theoretical basis for macro- and micro-flows can be used; nevertheless, certain effects can be of different importance for micro-systems if compared with macro-systems. Herwig and Hausner [3] called these effects "scaling effects with respect to a standard macro-analysis". Guo and Li [4] remarked that, since different forces have different length dependences, the surface forces (like surface tensions, viscous forces and electrostatic forces) become more important and even dominant as the scale is reduced. For this reason, it is possible to identify the following main "scaling effects" for single-phase flows in micro-channels: (i) axial heat conduction (small Peclet number); (ii) conjugate heat transfer; (iii) temperature dependent properties (in particular viscosity); (iv) wall roughness (non-uniform wall roughness distribution along the perimeter) and (v) viscous dissipation (high viscous forces).

Axial heat conduction and conjugate heat transfer in micro-channels have been studied numerically by several authors. The effects of thermal conductivity of the solid region around the micro-channels has been explored by Qu and Mudawar [5]; for the geometry investigated, the effects of the solid thermal conductivity on the average Nusselt number were so small that their results for a copper heat sink and a silicon heat sink were virtually identical. For the same geometry, Li et al. [6] observed that the magnitude of the heat flux due to axial heat conduction in the solid region depends on the Reynolds number. They concluded that for low fluid flow rates particular attention should be paid to the effects of this heat loss.

A number of papers have been devoted to analyze the effect of thermophysical property variations with temperature [7–10]. The main conclusion of these works is that the variation of the fluid viscosity cannot be neglected in the analysis of micro-flows, especially at low Reynolds numbers, whereas quantities such as density, specific heat etc. can be considered independent of temperature. This fact is, in part, linked to the different role of forces in micro-scales; mixed convection is unimportant in micro-channels because the values assumed by the Grashof number are very small (*Gr* is dependent on the third power of the characteristic length). That means that the classical Boussinesq approximation is not useful to study the convective heat transfer in micro-channels. On the contrary, since the viscous forces are important at the micro-scale, a new approximation can be proposed: to consider all the fluid properties as constant in the equations with the exception of the fluid viscosity.

The role of surface roughness on fluid flow and heat transfer in micro-channels has been emphasized by several authors both experimentally and theoretically [11–14]. Particularly interesting is the model proposed by Sabry [11] to explain the role of the surface roughness at the walls of a micro-channels. He stated that liquid flow is probably partially separated from the walls by a very thin gas blanket trapped by the roughness elements in micro-channels. This gas blanket could influence the value assumed by the friction factor, the critical Reynolds number that marks the transition from laminar-to-turbulent regimes and the Nusselt number and could explain the dependence of the friction factor and the Nusselt number on the Reynolds number even in the laminar regime.

The effect of viscous forces in micro-channels has also been investigated by many authors. Tso and Mahulikar [15,16] have made a theoretical and experimental analysis of circular micro-channels considering the effect of viscous dissipation by means of the Brinkman number. They found that the experimental data on the friction factor for laminar flow can be correlated well by using the Brinkman number. In their experimental analysis the authors found very small values of the Brinkman number (of the order of 10^{-8}), too low to affect directly the water bulk temperature by means of viscous dissipation. The author remarked that for micro-channels the effect of the Brinkman number is related to the reduction of the dynamic viscosity between the inlet and the outlet of a micro-channel due to the increase in the bulk temperature; this fact can reduce the Brinkman number at the exit by about 50% of its inlet value. The authors concluded that the axial variation of the Brinkman number affects the convective heat transfer in micro-channels; they defined this effect "the secondary effect" of the Brinkman number. The authors demonstrated that the secondary effect of the Brinkman number can be used to explain the decrease in the Nusselt number when the Reynolds number increases in the laminar regime.

Judy et al. [17] confirmed experimentally that viscous dissipation can be invoked to explain the deviation from Stokes flow behaviour repeated in many other works on micro-channels; they evidenced that viscous heating has the significant effect of increasing the temperature of the flowing fluid along the micro-channel axis.

Tunc and Bayazitoglu [18] studied the effects of viscous dissipation for rarefied gas flow in the slip-flow regime in circular and rectangular micro-channels. They concluded that the Brinkman number plays an important role in the heat transfer through micro-devices even for gas flows.

Xu et al. [19] investigated the viscous dissipation effects for liquid flows in micro-channels; they stated that deviations from predictions using conventional theory that neglects viscous dissipation could be expected because viscous dissipation tends to be significant due to the high velocity gradients existing in channels with small hydraulic diameters. They proposed a criterion to draw the limit of the significance of the viscous dissipation effects in micro-channel flows.

More recently, Koo and Kleinstreuer [20] investigated numerically the viscous dissipation effects on the temperature field and friction factor in circular and rectangular micro-channels. They demonstrated that viscous dissipation is strongly dependent on the hydraulic diameter and the channel aspect ratio. They concluded that ignoring viscous dissipation could affect accurate flow simulation in micro-channels.

In this paper, the explanation of the experimental results obtained for micro-channels in terms of friction factors will be researched within the conventional theory (Navier–Stokes equations) by considering in the analysis those effects that are not important in macro-scale but that become so when the dimensions of the system decrease (the scaling effects with respect to a standard macro-analysis). In particular, this paper focuses on the role of viscous heating in liquids flowing through micro-channels. When a fluid flows through a channel having a very small hydraulic diameter, the internal heat generation due to the viscous forces can produce a temperature rise even if the micro-channel is adiabatic. The temperature variation due to the viscous dissipation changes the values of the fluid thermophysical properties between inlet and outlet of the micro-channel. This effect becomes particularly significant for the viscosity of liquids flowing in micro-channels. In this paper, a criterion will be suggested to assess the significance of the effects of viscous dissipation in micro-channel flows. In order to highlight the influence of viscous forces, the other scaling effects will not be taken into account.

2. Mathematical model

Let us consider a micro-channel having an axially uniform cross-section with area equal to Ω and perimeter equal to Γ. The length of the channel is equal to L. An imposed linear heat flux q_w is present at the wall (H1 boundary condition). Some simplifying assumptions can be made before applying the conventional Navier–Stokes and energy equations to model the fluid flow and the heat transfer process in the micro-channel. The major assumptions are

1. the fluid is Newtonian, incompressible and with a laminar fully developed profile of velocity $u(\xi,\eta)$ and a uniform inlet temperature θ_{in};
2. the transport processes are considered to be steady-state and bi-dimensional (throughout the micro-channel the velocity and the temperature profiles are considered as fully developed);
3. thermal radiation is neglected;
4. all channel walls are rigid and non-porous;
5. axial thermal conduction ($Pe \gg 1$), natural convection ($Gr/Re^2 \ll 1$), and interior heat sources are neglected;
6. fluid thermophysical properties are assumed as constant.

Under the mentioned hypotheses the conservation equations of momentum and energy can be written as follows:

$$\begin{cases} \nabla^2 u = -\dfrac{1}{\mu}\dfrac{\mathrm{d}p}{\mathrm{d}\zeta}, \\ \nabla^2\theta + \dfrac{\mu}{k}[\nabla u \cdot \nabla u] = \dfrac{u(\xi,\eta)}{\alpha}\dfrac{\partial\theta}{\partial\zeta}. \end{cases} \quad (1)$$

In the fully developed thermal region of a heated duct the temperature profile continues to change with ζ but the "relative temperature shape" of the profile no longer changes. Using the definition of thermal fully developed region it is possible to demonstrate that, for the H1 boundary condition, the energy equation can be integrated on the area of the cross-section by obtaining the following result:

$$\frac{\partial\theta}{\partial\zeta} = \frac{\mathrm{d}\theta_b}{\mathrm{d}\zeta} = \frac{q_w + \mu\int_\Omega[\nabla u\cdot\nabla u]\,\mathrm{d}\Omega}{\rho c_p W \Omega}, \quad (2)$$

where q_w is the linear heat flux to the heated wall, W is the fluid average velocity, Ω is the area of the cross-section and θ_b is the bulk temperature defined as follows:

$$\theta_b = \frac{1}{\Omega}\int_\Omega u(\xi,\eta)\theta(\xi,\eta)\,\mathrm{d}\Omega. \quad (3)$$

It is suitable to introduce the following dimensionless quantities:

$$\begin{aligned} &x = \frac{\xi}{D_h}, \quad y = \frac{\eta}{D_h}, \quad z = \frac{\zeta}{L}, \\ &\Gamma^* = \frac{\Gamma}{D_h}, \quad \Omega^* = \frac{\Omega}{D_h^2}, \quad \nabla^* = D_h\nabla, \\ &V = \frac{u}{W}, \quad T = \frac{(\theta-\theta_{in})}{\Delta\theta_{ref}}, \quad L^* = \frac{L}{D_h}; \\ &p^* = -\frac{D_h^2}{\mu W}\frac{\mathrm{d}p}{\mathrm{d}\zeta}, \quad Ec = \frac{W^2}{2c_p\Delta\theta_{ref}}, \\ &Pe = \frac{WD_h}{\alpha} = Re\,Pr, \quad \tilde{q}_w = \frac{q_w}{k\Delta\theta_{ref}} \end{aligned} \quad (4)$$

Consequently, the dimensionless momentum and energy balance equations are readily obtained in the following forms for the problem examined:

$$\begin{cases} \nabla^{*2}V = -p^*, \\ \dfrac{\mathrm{d}T_b}{\mathrm{d}z} = \dfrac{\tilde{q}_w L^*}{Pe\Omega^*} + 2\dfrac{Ec\,L^*}{Re\,\Omega^*}\Phi^* \end{cases} \quad (5)$$

in which the dimensionless viscous-energy-dissipation function Φ^* is defined as

$$\Phi^* = \int_{\Omega^*}[\nabla^* V\cdot\nabla^* V]\,\mathrm{d}\Omega^*. \quad (6)$$

The momentum conservation equation is solved by using the no-slip boundary condition at the wall; which means considering $Kn < 0.001$ for liquid flows, so as to neglect any rarefaction effect. For a liquid flow, this assumption is justified; by assuming the typical mean free path λ of molecules under ambient conditions is in the range 0.1–1 nm, the Knudsen number ($Kn = \lambda/D_h$) can take values greater than 0.001 only if the hydraulic diameter of the micro-channel is less than 1 μm.

The complete set of boundary conditions is thus:

$$\begin{cases} V|_{\Gamma^*} = 0, \\ T_b|_{z=0} = 0. \end{cases} \quad (7)$$

By considering the thermophysical properties as constant, the momentum equation and the energy balance equation are uncoupled. From the velocity distribution $V(x,y)$ it is possible to derive the value assumed by the main flow parameters as a function of the micro-channel aspect ratio. In particular, it is possible to determine the value of the Poiseuille number, i.e. the product of the Fanning friction factor for fully developed flow (f) and the Reynolds number (Re), using the following relation:

$$f\,Re = -\frac{1}{2\Omega^*}\int_{\Omega^*}\nabla^*\cdot(\nabla^* V)\,\mathrm{d}\Omega^* = \frac{p^*}{2}. \quad (8)$$

It can be demonstrated that the Poiseuille number and the viscous-energy dissipation function Φ^* are related. Indeed, by using the properties of the Laplacian operator it is possible to write:

$$\nabla^* V\cdot\nabla^* V = -V\nabla^{*2}V + \tfrac{1}{2}\nabla^{*2}V^2. \quad (9)$$

By integrating Eq. (9) on the cross-section area of the micro-channel,

$$\int_{\Omega^*} \nabla^* V \cdot \nabla^* V \, d\Omega^* = -\int_{\Omega^*} V \nabla^{*2} V \, d\Omega^* + \frac{1}{2} \int_{\Omega^*} \nabla^{*2} V^2 \, d\Omega^*. \quad (10)$$

The second integral of the r.h.s. of Eq. (10) can be demonstrated to be zero:

$$\int_{\Omega^*} \nabla^{*2} V^2 \, d\Omega^* = \int_{\Gamma^*} n \cdot \nabla^* V^2 \, d\Gamma^* = 0. \quad (11)$$

By using the Navier–Stokes equation (Eq. (5)), the first term on the r.h.s. of Eq. (10) can be written as follows:

$$\int_{\Omega^*} V \nabla^{*2} V \, d\Omega^* = -p^* \int_{\Omega^*} V \, d\Omega^* = -p^* \Omega^* \quad (12)$$

By substituting Eqs. (11) and (12) in Eq. (10) it follows that:

$$\Phi^* = \int_{\Omega^*} \nabla^* V \cdot \nabla^* V \, d\Omega^* = 2 f Re \, \Omega^* = f Re \frac{\Gamma^*}{2}. \quad (13)$$

Eq. (13) states that the heat generation due to the viscous dissipation in the 1D average model obtained by integrating the energy equation on the cross-section can be lead back to the friction at the channel walls. By combining Eq. (13) with the energy equation in Eq. (5), it is possible to show how the axial variation of the bulk temperature is related to the Poiseuille number:

$$\frac{dT_b}{dz} = \frac{\tilde{q}_w L^*}{Pe \, \Omega^*} + 4 \frac{Ec}{Re} [f Re L^*]. \quad (14)$$

By integrating Eq. (14) from the inlet to the outlet of a long micro-channel, where the entrance effects are negligible, one obtains the temperature rise along the micro-channel:

$$T_b(z = 1) = \frac{\Delta\theta_b}{\Delta\theta_{ref}} = \frac{\tilde{q}_w L^*}{Pe \, \Omega^*} + 4 \frac{Ec}{Re} [f Re L^*]. \quad (15)$$

Two different contributions are present in Eq. (15); the first term on the r.h.s. represents the effect of the heat flux at the wall. The second term is the contribution to fluid heating due to viscous dissipation, the quantity in brackets depending on the geometry of the micro-channel only.

In heated or cooled macro-channels, the first term is predominant on the second term. It is easy to demonstrate that for macro-channels the second term is always negligible in the laminar regime. As an example, for a tube with ID equal to 1 cm and 1 m long in which water flows in laminar regime ($Re = 1000$) the temperature rise due to viscous heating is of the order of 10^{-5} K; for ID = 100 μm the temperature rise with the same conditions becomes of the order of 10 K. These examples evidence that the viscous heating can be considered a ''scaling effect'' for micro-channels.

In order to analyze the contribution of the viscous dissipation alone on the temperature rise, in the following discussion an adiabatic micro-channel will be considered.

By using Eq. (15) for an adiabatic micro-channel ($q_w = 0$), the value of the dimensionless temperature at the outlet of the micro-channel can be calculated as follows:

$$T_b(z = 1) = \frac{\Delta\theta_b}{\Delta\theta_{ref}} = 4 \frac{Ec}{Re} [f Re L^*]. \quad (16)$$

Hence, the outlet temperature increases if the mean velocity of the fluid increases and if the micro-channel hydraulic diameter decreases; the role of the micro-channel geometry is taken into account by means of the term in brackets of Eq. (16).

The temperature gradient along the micro-channel due to the viscous heating can be expressed as follows:

$$\frac{d\theta_b}{d\zeta} = \left(4 \frac{Ec}{Re} [f Re]\right) \frac{\Delta\theta_{ref}}{D_h}. \quad (17)$$

Eq. (17) is obtained by considering a constant value of the viscosity; in this manner, Eq. (17) gives the maximum value of the temperature rise related to the viscous heating, as demonstrated numerically by Koo and Kleinstreuer [20].

In order to determine the temperature difference between the inlet and the outlet of a micro-channel by means of Eq. (16) or (17) it is necessary to know the value assumed by the Poiseuille number in the laminar regime for the micro-channel considered. In this work, the values of the Poiseuille number for circular, rectangular, trapezoidal and double trapezoidal micro-channels determined numerically by Morini [21] are employed.

In Fig. 1 the cross-sections considered by Morini [21] are shown. The Poiseuille number for rectangular, trapezoidal and double-trapezoidal micro-channels depends on the aspect ratio. Two different definitions of the aspect ratio are used in literature for trapezoidal channels: $\gamma = h/a$ or $\beta = h/b$ with reference to the symbols shown in Fig. 1. For a $\langle 100 \rangle$ silicon micro-channel the aspect ratio γ cannot exceed the value of $tg(\phi)/2$ (equal to 0.707 for $\phi = 54.74°$) corresponding to the degeneration of the channel cross-section into a triangle.

The aspect ratio γ of a double-trapezoidal channel is defined as the ratio between the height (h) and the maximum width (a); γ can assume all the values between 0, the parallel plates configuration, and 1.414, the rhombic configuration.

Morini [21] gave a fifth order polynomial approximation for calculating $f Re$ as a function of the channel aspect ratio (γ) with the aim of offering a very simple but accurate tool for technicians and designers involved in micro-fluidic applications:

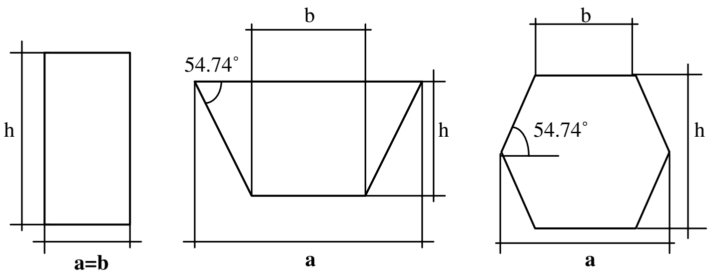

Fig. 1. Schematic representation of the most common silicon micro-channel cross-sections.

Table 1
Polynomial coefficients appearing in Eq. (18) for trapezoidal, double-trapezoidal and rectangular silicon micro-channels

	g_0	g_1	g_2	g_3	g_4	g_5	Δ (%)
Trapezoidal ⟨1 0 0⟩ *silicon micro-channel [21] (0 < γ < 0.707)*							
fRe	24	−42.267	64.272	−118.42	242.12	−178.79	−0.14
Double-trapezoidal ⟨1 0 0⟩ *silicon micro-channel [21] (0 < γ < 1.414)*							
fRe	24	−27.471	26.117	−6.6351	−0.2956	−0.5974	0.05
Rectangular ⟨1 1 0⟩ *silicon micro-channel [22] (0 < γ < 1)*							
fRe	24	−32.5272	46.7208	−40.8288	22.9536	−6.0888	0.05

$$f\,Re = \sum_{n=0}^{5} g_i \gamma^n. \tag{18}$$

A similar correlation has been presented by Shah and London [22] for rectangular ducts. The values of the constants g_i are listed in Table 1 for micro-channels having rectangular, trapezoidal and double-trapezoidal cross-sections; the maximum relative difference Δ quoted in Table 1 is positive when Eq. (18) gives values greater than the rigorous calculation.

3. Discussion of the results

In order to validate Eq. (17) the experimental data of Celata et al. [23,24] are used. Celata et al. [23] measured the temperature difference between the inlet and the outlet of a smooth capillary tube ($f\,Re = 16$) of fused silica with a hydraulic diameter equal to 101 μm through which water is circulated at different Reynolds numbers. The comparison between the prediction of Eq. (17) and the experimental data of Celata et al. [23,24] is shown in Fig. 2. It is possible to note that the agreement between the model and the measurements is good; in particular, the experimental values are generally lower than those predicted using the temperature gradient, as per Eq. (17). This fact can be explained by observing that Eq. (17) holds if the micro-tube can be considered adiabatic, whereas the micro-tube was not thermally insulated during the experimental tests. The agreement improves for larger Reynolds numbers; this fact is due to the smaller temperature rise at lower Reynolds number. In addition, it is possible to note that the micro-tubes tested by Celata et al. [23,24] were characterized by two different lengths ($L = 10.02$ cm and $L = 5.035$ cm) and the agreement between Eq. (17) and the experimental data is better for the longer micro-tubes. This is readily explained by observing that Eq. (17) has been obtained by neglecting the entrance effects and these effects become more

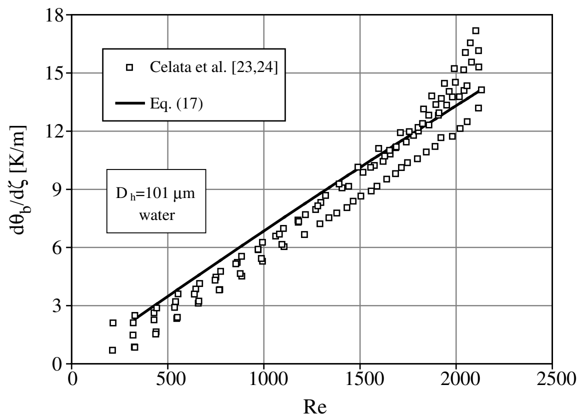

Fig. 2. Comparison between the temperature gradient predicted by Eq. (17) and the experimental data of Celata et al. [23,24] obtained with water in smooth fused silica micro-tubes having a hydraulic diameter of 101 μm.

important for the shorter micro-tubes. It is important to note that the value of the temperature gradient is not influenced by the choice of the reference temperature drop $\Delta\theta_{\mathrm{ref}}$ because this term is deleted by the same term in the Eckert number.

Fig. 3 depicts the value of the temperature gradient as a function of the hydraulic diameter for a fixed value of the Reynolds number ($Re = 300$) when water flows through a rectangular micro-channel. Two micro-channels with a rectangular cross-section ($\gamma = 0.1$ and $\gamma = 1$) are considered.

Judy et al. [17] for a square micro-channel with a hydraulic diameter of 74.1 μm (D_h), and 11.4 cm long (L) have experienced a temperature rise between the inlet and the outlet of 6.2 °C when *iso*-propanol was employed as working fluid with $Re = 300$; in the same conditions, by using Eq. (16) one obtains a temperature rise of 5.8 °C. The thermophysical properties as functions of temperature used in the present work for water, methanol and *iso*-propanol are from [25].

In Fig. 3 it is evident that for different fluids the viscous dissipation effect can play different roles. By using Eq. (17) it is possible to note that, for two different fluids, c and d, and for a fixed value of the Reynolds number, hydraulic diameter and cross-section geometry, the temperature gradient ratio is linked to the following ratio of the fluid thermophysical properties:

$$\frac{\left(\frac{\mathrm{d}\theta_b}{\mathrm{d}\zeta}\right)_c}{\left(\frac{\mathrm{d}\theta_b}{\mathrm{d}\zeta}\right)_d} = \left(\frac{\nu_c}{\nu_d}\right)^2\left(\frac{c_{pd}}{c_{pc}}\right). \tag{19}$$

The temperature gradient due to the viscous heating is larger for fluid having a large value of the kinematic viscosity and a low value of the specific heat. For example, for *iso*-propanol and water this ratio is equal to 13.26 at 298 K; for methanol and water this ratio is equal to 2.32 at 298 K.

In Fig. 3 the comparison between the values of the temperature gradient for *iso*-propanol and water are shown; as expected, the viscous heating, for a fixed value of the Reynolds number of the hydraulic diameter and micro-channel cross-section, is stronger when *iso*-propanol is used as working fluid.

In addition, it is well evident that the effects of the viscous dissipation are negligible for $Re = 300$ for micro-channels having a hydraulic diameter larger than 300 μm.

It is interesting to note that, for water and air the ratio quoted in Eq. (19) is equal to 10^3 at 298 K; this means that the effects of the viscous dissipation could be very strong for air. However, the influence of compressibility is very important for gases flowing through micro-channels; the gas expansion in the flow direction causes the temperature to decrease and it contrasts the effects of viscous heating. In other words, for gases the constant properties model presented in this paper can be useful only if these two conditions are verified:

$$Ma < 0.2 \quad \frac{\Delta p}{p_{\mathrm{in}}} < 0.05, \tag{20}$$

where Ma is the average Mach number ($=W/c$) between the inlet and outlet of the micro-channel, Δp is the pressure drop through the micro-channel and p_{in} is the initial static pressure. In general, for micro-channels the second condition is the stricter constraint.

By observing Fig. 3, it is evident that the effects of the viscous dissipation are more important for decreasing aspect ratios of the micro-channel. This feature can be further evidenced by using the data of Fig. 4 where the

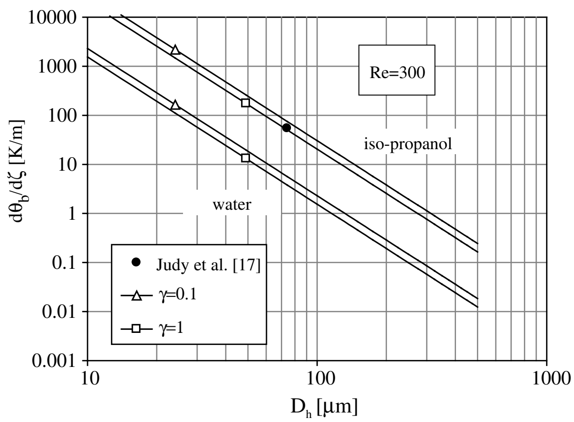

Fig. 3. Temperature gradient as a function of the hydraulic diameter for square ($\gamma = 1$) and rectangular ($\gamma = 0.1$) micro-channels for water at $Re = 300$.

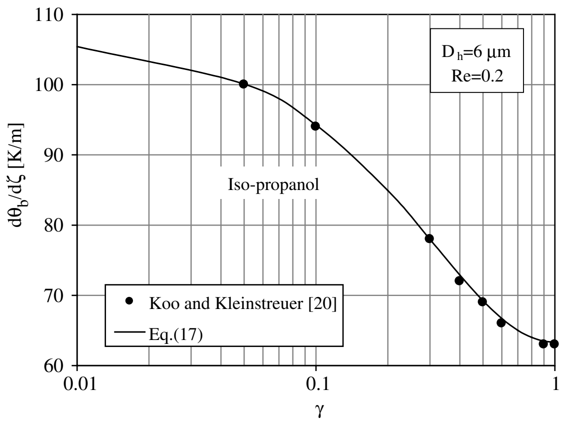

Fig. 4. The effect of the aspect ratio on the temperature gradient due to viscous dissipation in a rectangular micro-channel: comparison with the numerical results obtained by Koo and Kleinstreuer [20].

value of the temperature gradient related to viscous heating in a rectangular micro-channel for *iso*-propanol with a fixed value of the Reynolds number ($Re = 0.2$) and of the hydraulic diameter ($D_h = 6$ μm) is plotted as a function of the channel aspect ratio (γ). The numerical results obtained by Koo and Kleinstreuer [20] in the same conditions are utilized to prove the reliability of the results obtained by means of Eq. (17).

The dependence of the viscous dissipation on the Reynolds number in rectangular micro-channels is evidenced in Fig. 5; a square micro-channel having a hydraulic diameter of 74 μm is considered in order to compare the results obtained by means of Eq. (17) with the numerical results of Koo and Kleinstreuer [20]. The viscous dissipation clearly increases with the square of the Reynolds number; the difference between the numerical results obtained by Koo and Kleinstreuer tends to increase for high Reynolds number; this fact is due to the effects of the entrance region which are not considered in Eq. (17). The results of Koo and Kleinstreuer [20] put in evidence the entrance effects on the temperature rise; by using these results, it is possible to conclude that the hypothesis of a constant temperature gradient along the micro-channel holds in the large part of the micro-channel, especially for low Reynolds numbers.

In Fig. 6 the temperature gradient due to viscous heating is shown as a function of the aspect ratio for trapezoidal, rectangular and double trapezoidal cross-sections. The working fluid is *iso*-propanol and fixed values of the Reynolds number ($Re = 300$) and of the hydraulic diameter (40 μm) are considered. The viscous heating increases when the aspect ratio decreases; in a trapezoidal micro-channel 1 cm long, the temperature rise between the inlet and the outlet is equal to 5 °C for $\gamma = 0.05$; if the trapezoidal cross-section degenerates in a triangular cross-section ($\gamma = 0.707$) the temperature rise becomes equal to 3 °C. It is interesting to note that

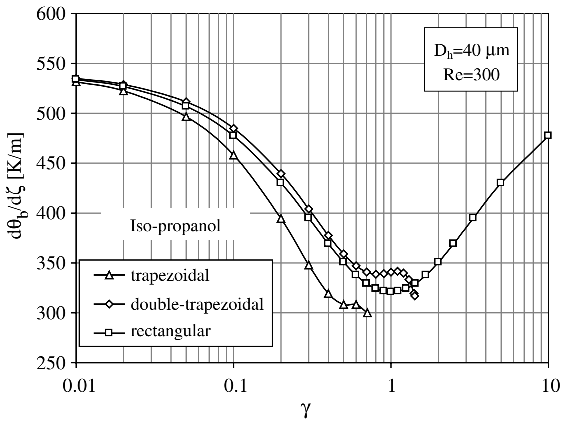

Fig. 6. Viscous heating as a function of the aspect ratio of the typical cross-sections of silicon micro-channels.

the viscous heating is stronger in double-trapezoidal for a fixed value of the aspect ratio. This fact can be explained by considering the number of corners of the cross-section. For low values of the aspect ratio the influence of the geometry on the temperature gradient becomes negligible.

In Fig. 7 the temperature rises obtained by Koo and Kleinstreuer [20] for *iso*-propanol with low Reynolds numbers ($Re < 1$) in order to reproduce the experimental data obtained for trapezoidal micro-channels by Pfhaler et al. [26] are compared with the prediction of Eq. (16). Koo and Kleinstreuer simulated the trapezoidal micro-channels as rectangular ducts with the same hydraulic diameter and aspect ratio; in this work this approximation is removed. The geometrical characteristics of the four channels tested by Pfhaler [26] are summarized in Table 2.

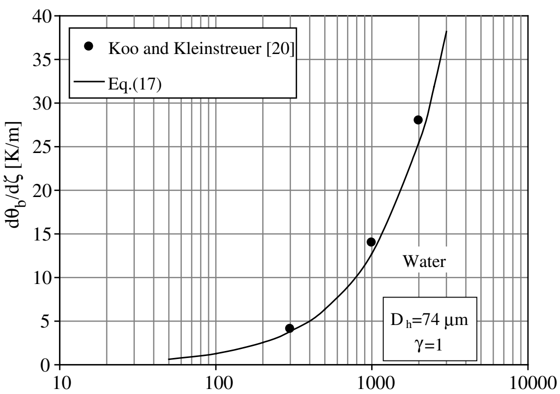

Fig. 5. The effect of the Reynolds number on the temperature gradient due to the viscous heating for water in a square micro-channel with a hydraulic diameter equal to 74.1 μm.

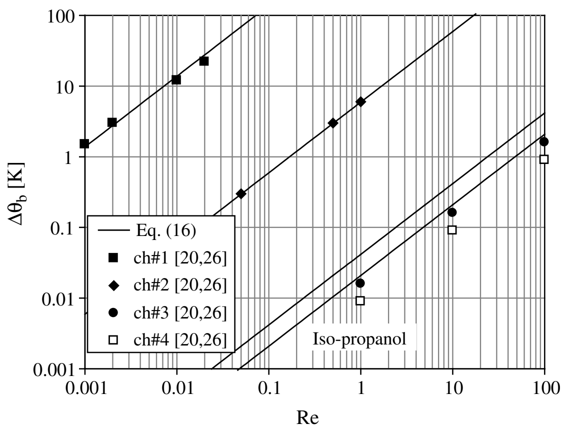

Fig. 7. Temperature rise due to the viscous dissipation along the silicon trapezoidal micro-channels tested by Pfhaler et al. [26].

Table 2
Maximum width (a), height (h), length (L) and hydraulic diameter of the ⟨1 0 0⟩ silicon trapezoidal micro-channels tested by Pfhaler et al. [26]

	a [μm]	h [μm]	L [m]	D_h [μm]
ch#1	115	0.48	0.0105	0.96
ch#2	110	3	0.0105	5.80
ch#3	86.25	24.4	0.0109	34.05
ch#4	50	38.7	0.0102	25.02

By observing Fig. 7 it is possible to note that even in the very low Reynolds number range the effects of viscous heating can be very strong if the hydraulic diameter of the micro-channel is small. The agreement with the data of Koo and Kleinstreuer [20] is good for ch#1, ch#2 and ch#3; on the contrary, the present results for ch#4 are quite different from the results obtained by these authors. This fact is due to the approximation used, i.e. considering the trapezoidal cross-section as rectangular with the same aspect ratio. As demonstrated in Fig. 6, for low aspect ratio, trapezoidal and rectangular micro-channels exhibit a similar behaviour; this fact explains the agreement experienced when ch#1 ($\gamma = 0.004$), ch#2 ($\gamma = 0.027$) and ch#3 ($\gamma = 0.28$) are considered. On the contrary, for ch#4 the cross-section is triangular and the aspect ratio is maximum ($\gamma = 0.707$); in this case the approximation used by Koo and Kleinstreuer does not hold. This fact confirms the important role of the cross-sectional geometry on viscous heating.

Judy et al. [17] have shown that the experimental Poiseuille numbers are strongly influenced by the viscosity variation with temperature along the micro-channel. They calculated $f\,Re$ in two different ways: (i) by using a viscosity based on the inlet temperature and (ii) using a viscosity based on the average temperature between the inlet and the outlet. When the temperature at the inlet was used to evaluate the viscosity, the friction factor decreased with increasing Reynolds number as the effect of viscous heating becomes more pronounced at higher velocities. They demonstrated that the theoretical value of the Poiseuille number ($f\,Re_{th}$), independent of Reynolds number in laminar regime, and the experimental Poiseuille number, in which the viscosity is calculated by using the inlet temperature ($f\,Re_{exp}$), can be related through:

$$f\,Re_{exp} = \left(\frac{\mu(\theta_m)}{\mu(\theta_{in})}\right) f\,Re_{th}, \tag{21}$$

where θ_m is the mean temperature between the inlet and the outlet.

By using Eq. (16) to calculate the temperature rise due to the viscous heating in a square micro-channel ($D_h = 74.1$ μm) in which *iso*-propanol is used as working fluid and keeping in mind that for a square channel the theoretical Poiseuille number is equal to 14.227, it is possible to predict the value of the experimental Poiseuille number as a function of the Reynolds number and compare these results with the experimental results obtained by Judy et al. [17].

Fig. 8 shows how the agreement between the experimental values of $f\,Re$ found by Judy et al. can be correctly predicted by using the theory developed in this paper on the role of the viscous heating in micro-channels. This fact demonstrates that the viscous heating can explain some experimental observations in which a reduction of the friction factor in the laminar regime as the Reynolds number increases was reported.

It is possible to suggest a criterion to establish when viscous dissipation effects cannot be ignored in micro-channels. If one considers the temperature gradient in the flow direction as a constant, Eq. (16) gives the fluid temperature rise between the inlet and the outlet due to the viscous dissipation in an adiabatic micro-channel.

In Eq. (16) $\Delta\theta_{ref}$ is a reference temperature rise; this value can be fixed by considering the temperature sensitivity of fluid viscosity; for instance, it could be defined as the temperature rise for which the dynamic viscosity decreases about 2–3%. For water and *iso*-propanol the dynamic viscosity decreases about 20–25% per 10 K; for this reason, the $\Delta\theta_{ref}$ can be assumed equal to 1 K.

In order to neglect the effects of the viscous dissipation in adiabatic flows through micro-channels, the temperature rise due to viscous dissipation between the inlet and the outlet shall be less than the reference temperature rise; in other words, the viscous dissipation effects in adiabatic micro-channels cannot be neglected if the following condition is satisfied:

$$4\frac{Ec}{Re}[L^{*} f\,Re] \geqslant 1. \tag{22}$$

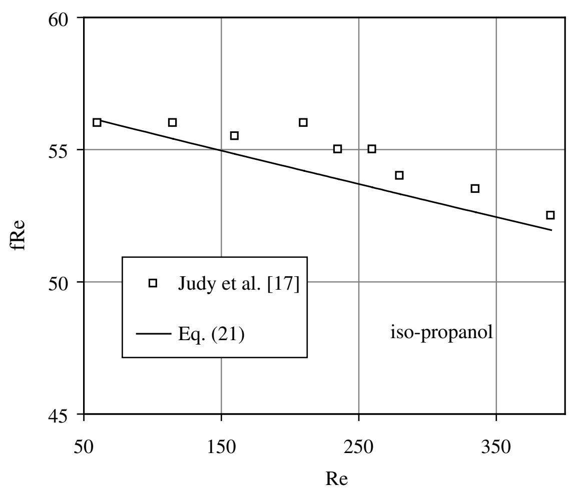

Fig. 8. Comparison between the experimental values of the Poiseuille number obtained by Judy et al. [17] and Eq. (21).

Eq. (22) can be adopted as a criterion to predict the upper limit of significance of viscous dissipation in a micro-channel. Eq. (22) allows for a fixed micro-channel geometry and hydraulic diameter the calculation of the values of the Reynolds number for which the temperature rise between inlet and outlet is equal to or greater than $\Delta\theta_{\text{ref}}$.

Fig. 9 plots the Reynolds number in correspondence of which the temperature rise, between inlet and outlet of a micro-tube 5 cm long, becomes equal to 1 K ($\Delta\theta_{\text{ref}}$) as a function of the hydraulic diameter of the micro-channel, for both water and *iso*-propanol flows.

The criterion based on Eq. (22) has been compared with the criterion proposed by Xu et al. [19] to draw the limit of significance for viscous dissipation effects in micro-channels.

They suggested that the limit of the viscous dissipation effects has to be linked to a temperature rise of 1 K between inlet and outlet. From the Buckingham π theorem they derived that the aforementioned criterion takes the form:

$$\left[\frac{\mu W^2 L}{\rho W c_p \theta_{\text{ref}} D_{\text{h}}^2}\right] Pr^{-0.1} > 0.056. \tag{23}$$

The authors called Viscous number (Vi) the term in brackets and deduced the coefficients in this correlation by using their numerical results for water through circular micro-channels.

In Fig. 9 the values of the minimum Reynolds number obtained by using the proposed criterion (Eq. (22)) and those calculated by using the criterion proposed by Xu et al. [19] are compared for water and *iso*-propanol (by using $\theta_{\text{ref}} = 1$ K as suggested by the authors). The criterion proposed by Xu et al. tends to overestimate the effects of the viscous dissipation with respect to Eq. (22).

In Fig. 10 the role of the cross-sectional geometry on the minimum Reynolds number calculated by means of

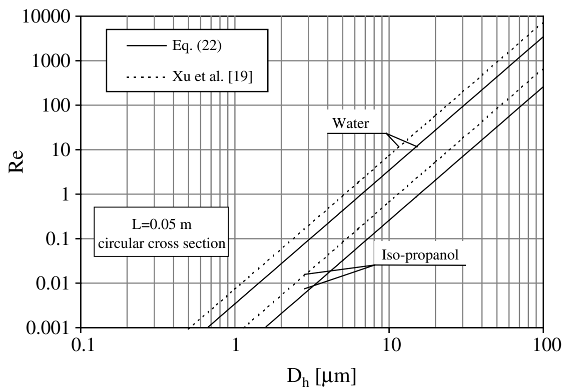

Fig. 9. Reynolds numbers for which the temperature rise due to the viscous heating along a circular micro-channel 5 cm long is equal to 1 K ($\Delta\theta_{\text{ref}}$) as a function of the hydraulic diameter.

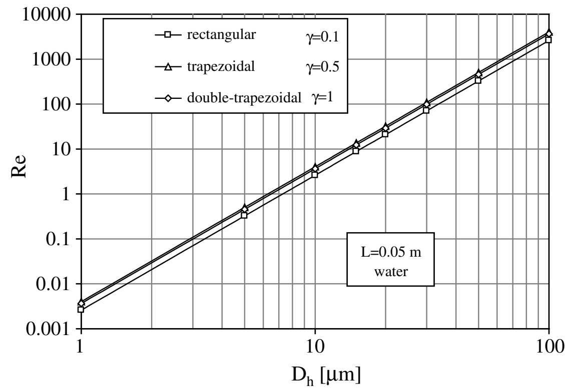

Fig. 10. Reynolds numbers for which the temperature rise due to the viscous heating in a micro-channel 5 cm long is equal to 1 K ($\Delta\theta_{\text{ref}}$) as a function of the hydraulic diameter for rectangular ($\gamma = 0.1$), trapezoidal ($\gamma = 0.5$) and double trapezoidal ($\gamma = 1$) cross-section.

Eq. (22) is shown for rectangular ($\gamma = 0.1$), trapezoidal ($\gamma = 0.5$) and double-trapezoidal ($\gamma = 1$) micro-channels. It is shown that the viscous dissipation becomes significant early for shallow cross-sections (low aspect ratio γ).

Finally, it must be noted that Eq. (16) can be employed to determine experimentally the value assumed by the apparent friction factor:

$$f_{\text{app}} = \frac{\Delta\theta_{\text{b}}}{\Delta\theta_{\text{ref}}}\left[\frac{1}{4EcL^*}\right]. \tag{24}$$

By using Eq. (24), the friction factor can be determined without measuring the pressure drop along the micro-channel but by means of temperature and flow rate measurement alone. This kind of measurement is not suitable when macro-channels are tested: for this reason Eq. (24) can be considered as an example of the role of scaling effects and to suggest new measurement procedures at the micro-scales.

A future work based on experimental results obtained for adiabatic micro-channels will be devoted to demonstrate how Eq. (24) can be used to determine the friction factors in very small micro-channels ($D_{\text{h}} < 100$ μm) and to identify the critical Reynolds number that marks the laminar-to-turbulent transition in micro-channels.

4. Conclusions

Often "unexpected experimental results" obtained by testing liquid flows through micro-channels are actually attributed to scaling effects and/or to the lack of accuracy in the experimental tests. By integrating the conventional theory with the effects that tend to become important in micro systems it is possible to explain many experimental results reported in the open literature without invoking "new micro-effects". The viscous dissipation effect was demonstrated to be a typical "scaling effect" for micro-channel flows; this effect can become very important for liquid flows when the hydraulic diameter is less than 100 μm. Based on the conventional theory, a model to predict the viscous dissipation effects in a micro-channel with an axially unchanging cross-section was developed. The temperature rise in an adiabatic micro-channel has been expressed as a function of the Eckert, Reynolds and Poiseuille numbers. Numerical and experimental results that appeared in the open literature were used as a benchmark in order to verify the reliability of the proposed model. After that, the role of the fluid thermophysical properties and of the micro-channel cross-section geometry on the viscous dissipation was analyzed and discussed.

By taking into account the viscous heating and the consequent decrease of the fluid viscosity, it has been shown that it is possible to explain the decrease of the friction factor when the Reynolds number increases as observed by some researchers. In addition, a criterion has been presented to discern the limit for which the effect of viscous dissipation can no longer be regretted in micro-channels.

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